Motor-vehicle dynamics

Dynamics of linear motion

Symbol		Unit
A	Largest cross-section of vehicle 1)	m2
a	Acceleration, braking (deceleration)	m/s2
cw	Drag coefficient	–
F	Motive force	N
Fcf	Centrifugal force	N
FL	Aerodynamic drag	N
FRo	Rolling resistance	N
FSt	Climbing resistance	N
Fw	Running resistance	N
f	Coefficient of rolling resistance	–
G	Weight = m · g	N
GB	Sum of wheel forces on driven or braked wheels	N
g	Gravitational acceleration = 9.81 m/s2 ≈ 10 m/s2	m/s2
i	Gear or transmission ratio between engine and drive wheels	–
M	Engine torque	N · m
m	Vehicle mass (weight)	kg
n	Engine speed	min–1
P	Power	W
Pw	Motive power	W
p	Gradient (= 100 tan α)	%
r	Dynamic radius of tire	m
s	Distance traveled	m
t	Time	s
υ	Vehicle speed	m/s
υ0	Headwind speed	m/s
W	Work	J
α	Gradient angle	°
μr	Coefficient of static friction	–

Additional symbols and units in text.

1) On passenger cars A ≈ 0.9 × Track × Height.

Total running resistance

The running resistance is calculated as:

FW = FRo + FL + FSt

Running-resistance power

The power which must be transmitted through the drive wheels to overcome running resistance is:

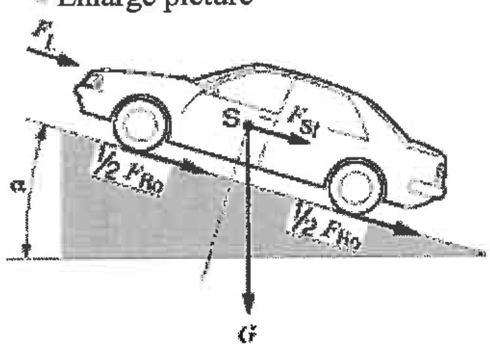
PW = FW · υ

or

$$P_W = \frac{F_W \cdot \upsilon}{3600}$$

with PW in kW, FW in N, υ in km/h.

Enlarge picture



Running resistance

Rolling resistance

The rolling resistance FRo is the product of deformation processes which occur at the contact patch between tire and road surface.

FRo = f · G = f · m · g

An approximate calculation of the rolling resistance can be made using the coefficients provided in the following table and in the diagram.

Road surface	Coefficient of rolling resistance f
Pneumatic car tires on	
Large sett pavement	0.015
Small sett pavement	0.015
Concrete, asphalt	0.013
Rolled gravel	0.02
Tarmacadam	0.025

Unpaved road	0.05
Field	0.1...0.35
Pneumatic truck tires on concrete, asphalt	0.006...0.01
Strake wheels in field	0.14...0.24
Track-type tractor in field	0.07...0.12
Wheel on rail	0.001...0.002

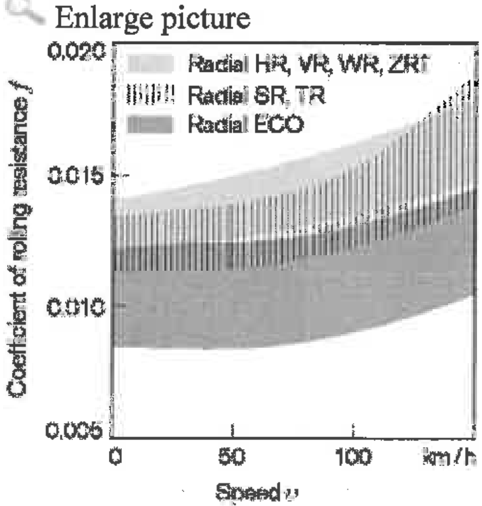
The increase in the coefficient of rolling resistance f is directly proportional to the level of deformation, and inversely proportional to the radius of the tire. The coefficient will thus increase in response to greater loads, higher speeds and lower tire pressure.

During cornering, the rolling resistance is augmented by the cornering resistance

FK = fK · G

The coefficient of cornering resistance fK is a function of vehicle speed, curve radius, suspension geometry, tires, tire pressure, and the vehicle's response under lateral acceleration.

Enlarge picture



Rolling resistance of radial car tires on smooth, level road surfaces under normal load and at correct tire pressure

Aerodynamic drag

Aerodynamic drag is calculated as:

$$F_L = 0.5 \cdot \rho \cdot c_w \cdot A\, (\upsilon + \upsilon_0)^2$$

With υ in km/h:

$$F_L = 0.0386 \cdot \rho \cdot c_w \cdot A \cdot (\upsilon + \upsilon_0)^2$$

Air density ρ (at 200 m altitude $\rho = 1.202$ kg/m3),

Drag coefficient c_w see Basic equations and various body configurations.

Aerodynamic drag in kW

$$P_L = F_L \cdot \upsilon = 0.5 \cdot \rho \cdot c_w \cdot A \cdot \upsilon \cdot (\upsilon + \upsilon_0)^2$$

or

$$P_L = 12.9 \cdot 10^{-6} \cdot c_w \cdot A \cdot \upsilon \cdot (\upsilon + \upsilon_0)^2$$

with P_L in kW, F_L in N, υ and υ_0 in km/h, A in m2, $\rho = 1.202$ kg/m3.

Empirical determination of coefficients for aerodynamic drag and rolling resistance

Allow vehicle to coast down in neutral under windless conditions on a level road surface. The time that elapses while the vehicle coasts down by a specific increment of speed is measured from two initial velocities, υ_1 (high speed) and υ_2 (low speed). This information is used to calculate the mean deceleration rates a_1 and a_2. See following table for formulas and example.

The example is based on a vehicle weighing m = 1450 kg with a cross section A = 2.2 m2.

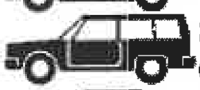
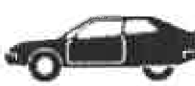
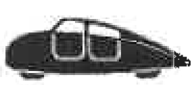
The method is suitable for application at vehicle speeds of less than 100 km/h.

	1st Trial (high speed)	2nd Trial (low speed)
Initial velocity	$\upsilon_{a1} = 60$ km/h	$\upsilon_{a2} = 15$ km/h
Terminal velocity	$\upsilon_{b1} = 55$ km/h	$\upsilon_{b2} = 10$ km/h
Interval between υ_a and υ_b	$t_1 = 6.5$ s	$t_2 = 10.5$ s
Mean velocity	$\upsilon_1 = \frac{\upsilon_{a1} + \upsilon_{b1}}{2} = 57{,}5$ km/h	$\upsilon_2 = \frac{\upsilon_{a2} + \upsilon_{b2}}{2} = 12{,}5$ km/h
Mean deceleration	$a_1 = \frac{\upsilon_{a1} - \upsilon_{b1}}{t_1} = 0{,}77 \frac{km/h}{s}$	$a_2 = \frac{\upsilon_{a2} - \upsilon_{b2}}{t_2} = 0{,}48 \frac{km/h}{s}$

Drag coefficient $c_W = \dfrac{6\,m \cdot (a_1 - a_2)}{A \cdot (v_1^2 - v_2^2)} = 0{,}36$

Coefficient of rolling resistance $f = \dfrac{28{,}2\,(a_2 \cdot v_1^2 - a_1 \cdot v_2^2)}{10^3 \cdot (v_1^2 - v_2^2)} = 0{,}013$

Drag coefficient and associated power requirements for various body configurations

	Drag coefficient cW	Drag power in kW, average values for A = 2 m2 at various speeds1) 40 km/h	80 km/h	120 km/h	160 km/h
 Open convertible	0.5...0.7	1	7.9	27	63
 Station wagon (2-box)	0.5...0.6	0.91	7.2	24	58
 Conventional form (3-box)	0.4...0.55	0.78	6.3	21	50
 Wedge shape, headlamps and bumpers integrated into body, wheels covered, underbody covered, optimized flow of cooling air.	0.3...0.4	0.58	4.6	16	37
 Headlamps and all wheels enclosed within body, underbody covered	0.2...0.25	0.37	3.0	10	24
 Reversed wedge shape (minimal cross-section at tail)	0.23	0.38	3.0	10	24
 Optimum streamlining	0.15...0.20	0.29	2.3	7.8	18
Trucks, truck-trailer combinations	0.8...1.5	–	–	–	–
Motorcycles	0.6...0.7	–	–	–	–
Buses	0.6...0.7	–	–	–	–
Streamlined buses	0.3...0.4	–	–	–	–

1) No headwind ($\upsilon 0 = 0$).

Climbing resistance and downgrade force

Climbing resistance (FSt with positive operational sign) and downgrade force (FSt with negative operational sign) are calculated as:

$$FSt = G \cdot \sin\alpha = m \cdot g \cdot \sin\alpha$$

or, for a working approximation:

$$FSt \approx 0.01 \cdot m \cdot g \cdot p$$

valid for gradients up to $p \leq 20$ %, as $\sin\alpha \approx \tan\alpha$ at small angles (less than 2 % error).

Climbing power is calculated as:

$$PSt = FSt \cdot \upsilon$$

with PSt in kW, FSt in N, υ in km/h:

$$P_{St} = \frac{F_{St} \cdot \upsilon}{3600} = \frac{m \cdot g \cdot \upsilon \cdot \sin\alpha}{3600}$$

or, for a working approximation:

$$P_{St} \approx \frac{m \cdot g \cdot p \cdot \upsilon}{360\,000}$$

The gradient is:

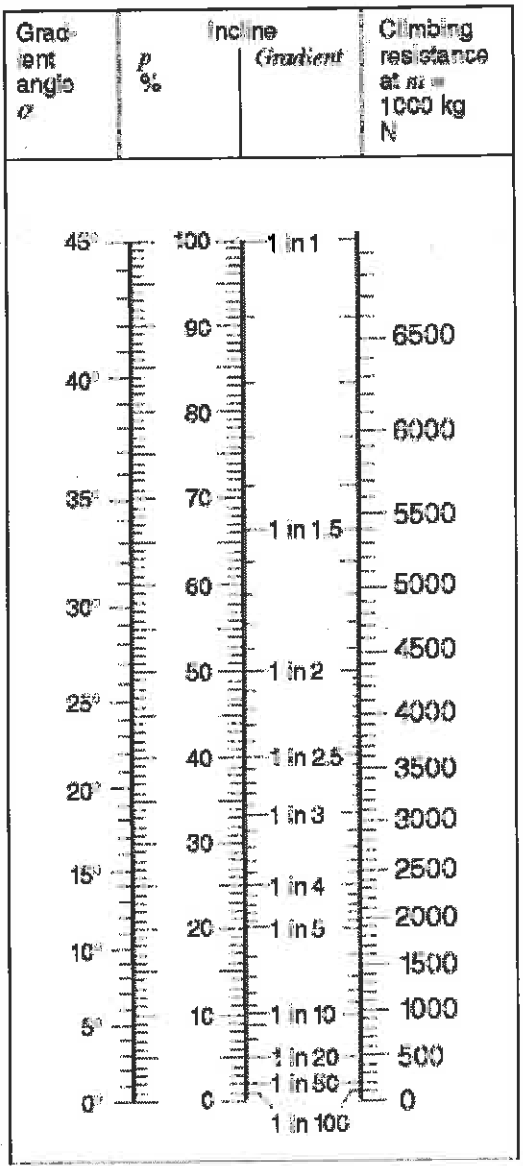
$$p = (h/l) \cdot 100\ \%\ \text{or}$$

$$p = (\tan\alpha) \cdot 100\ \%$$

with h as the height of the projected distance l. In English-speaking countries, the gradient is calculated as follows:

Conversion:

Gradient 1 in 100/p



Example: "1 in 2".

Values at m = 1000 kg

Climbing resistance FSt N	Climbing power PSt in kW at various speeds 20 km/h	30 km/h	40 km/h	50 km/h	60 km/h
6500	36	54	72	–	–
6000	33	50	67	–	–
5500	31	46	61	–	–
5000	28	42	56	69	–
4500	25	37	50	62	–
4000	22	33	44	56	67
3500	19	29	39	49	58
3000	17	25	33	42	50
2500	14	21	28	35	42

2000	11	17	22	28	33
1500	8.3	12	17	21	25
1000	5.6	8.3	11	14	17
500	2.3	4.2	5.6	6.9	8.3
0	0	0	0	0	0

Example: To climb a hill with a gradient of p = 18 %, a vehicle weighing 1500 kg will require approximately $1.5 \cdot 1700$ N = 2550 N motive force and, at $\upsilon = 40$ km/h, roughly $1.5 \cdot 19$ kW = 28.5 kW climbing power.

Motive force

The higher the engine torque M and overall transmission ratio i between engine and driven wheels, and the lower the power-transmission losses, the higher is the motive force F available at the drive wheels.

$$F = \frac{M \cdot i}{r} \cdot \eta$$

or

$$F = \frac{P \cdot \eta}{\upsilon}$$

η Drivetrain efficiency level
(longitudinally installed engine $\eta \approx 0.88...0.92$)
(transverse engine $\eta \approx 0.91...0.95$)

The motive force F is partially consumed in overcoming the running resistance FW. Numerically higher transmission ratios are applied to deal with the substantially increased running resistance encountered on gradients (gearbox).

Vehicle and engine speeds

$$n = \frac{60 \cdot \upsilon \cdot i}{2 \cdot \pi \cdot r}$$

or with υ in km/h:

$$n = \frac{1000 \cdot v \cdot i}{2 \cdot \pi \cdot 60 \cdot r}$$

Acceleration

The surplus force F – FW accelerates the vehicle (or retards it when FW exceeds F).

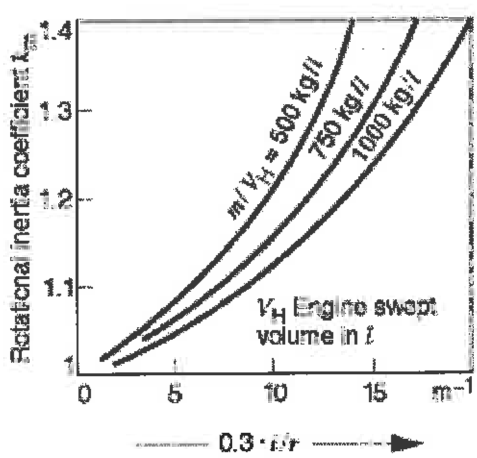
$$a = \frac{F - F_W}{k_m \cdot m}$$

or

$$a = \frac{P \cdot \eta - P_W}{v \cdot k_m \cdot m}$$

The rotational inertia coefficient km, compensates for the apparent increase in vehicle mass due to the rotating masses (wheels, flywheel, crankshaft, etc.).

Enlarge picture

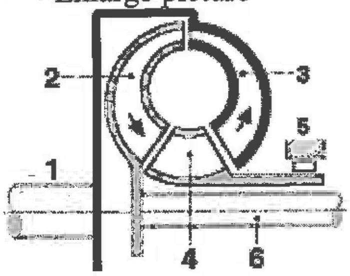


Determining the rotational inertia coefficient km

Motive force and road speed on vehicles with automatic transmissions

When the formula for motive force is applied to automatic transmissions with hydrodynamic torque converters or hydrodynamic clutches, the engine torque M is replaced by the torque at the converter turbine, while the rotational speed of the converter turbine is used in the formula for engine speed.

Enlarge picture



Hydrodynamic converter
1 Power, 2 Turbine, 3 Pump, 4 Stator, 5 One-way clutch, 6 Output.

The relationship between MTurb = f (nTurb) and the engine characteristic MMot = f (nMot) is determined using the characteristics of the hydrodynamic converter (see Retarder braking systems).

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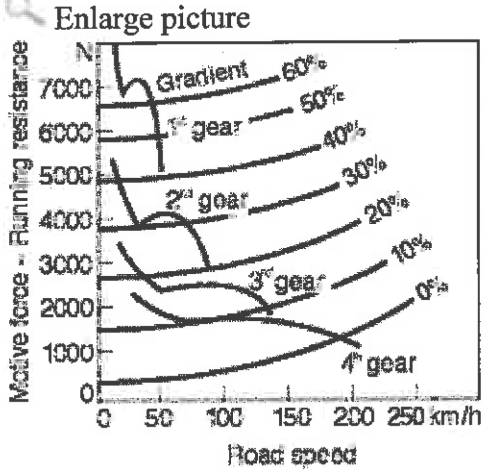

Running diagram for car with automatic transmission and hydrodynamic trilok converter under full throttle